GEOMETRÍA

PLANA DE GEOMETRÍA

CICLO REPASO UNI

ACADEMIA CÉSAR VALLEJO

PRÁCTICA N° 3

- ❑ RELACIONES MÉTRICAS II
- ❑ POLÍGONOS REGULARES
- ❑ ÁREAS DE REGIONES PLANAS I

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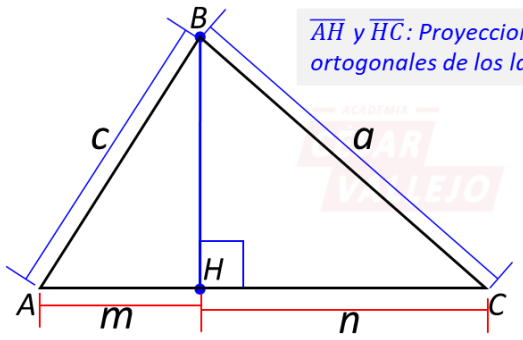
RELACIONES MÉTRICAS II

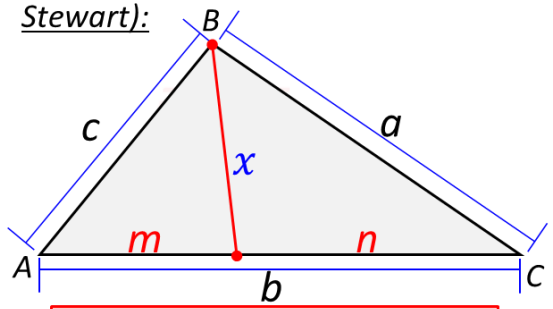
TEOREMA DE PROYECCIONES:



$\overline{AH}$ y $\overline{HC}$: Proyecciones ortogonales de los lados.

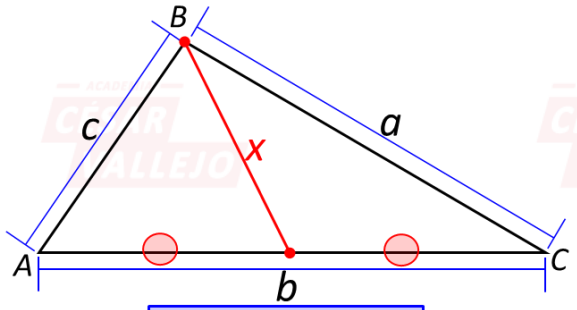
$$a^2 - c^2 = n^2 - m^2$$

CÁLCULOS DE LA CEVIANA (teorema de Stewart):



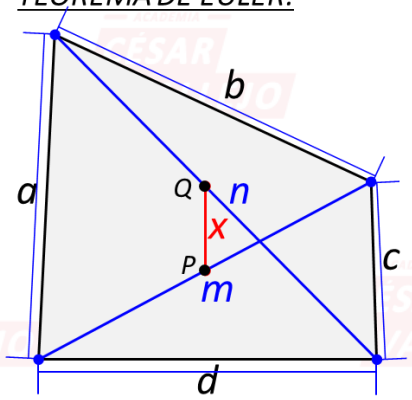
$$(a)^2m + (c)^2n = (x)^2b + bmn$$

CÁLCULO DE LA MEDIANA:



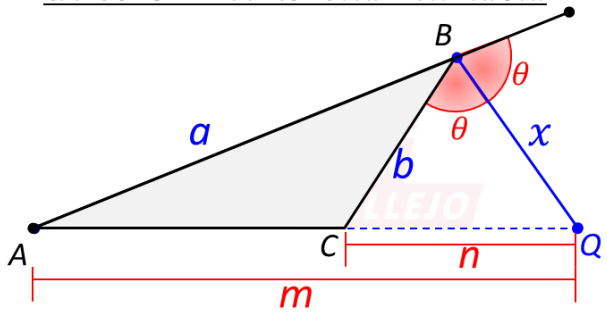
$$a^2 + c^2 = 2x^2 + \frac{b^2}{2}$$

TEOREMA DE EULER:



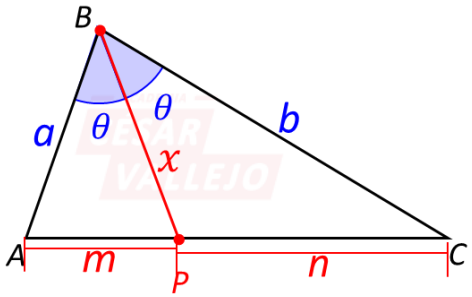
$$(a)^2+(b)^2+(c)^2+(d)^2=(m)^2+(n)^2+4(x)^2$$

CÁLCULO DE LA BISECTRIZ EXTERIOR:



$$x^2 = (m)(n) - (a)(b)$$

CÁLCULO DE LA BISECTRIZ INTERNA:

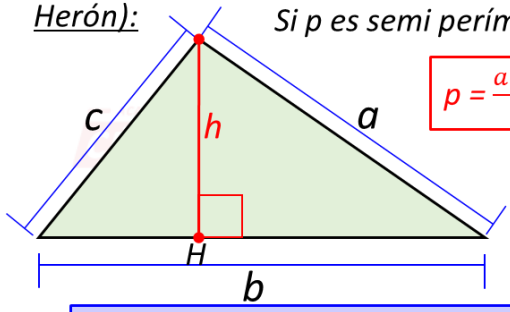


$$x^2 = (a)(b) - (m)(n)$$

CÁLCULO DE LA ALTURA (Teorema de Herón):

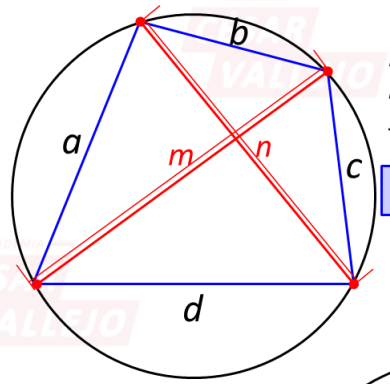
Si p es semi perímetro

$$p = \frac{a+b+c}{2}$$



$$h = \frac{2}{b}\sqrt{p(p-a)(p-b)(p-c)}$$

TEOREMA DE PTOLOMEO:

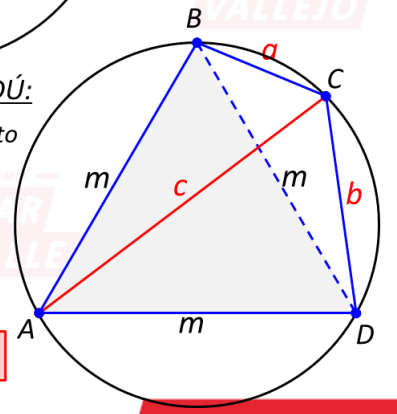


En todo cuadrilátero inscrito o inscriptible se cumple:

$$(m)(n)=(a)(c)+(b)(d)$$

TEOREMA DE CHADÚ:

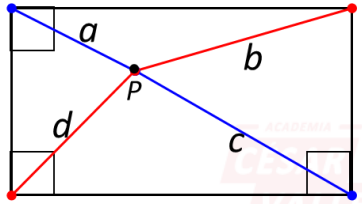
En un cuadrilátero inscrito o inscriptible donde ABD es equilátero.



$$c = a + b$$

$$2m^2 = a^2 + b^2 + c^2$$

TEOREMA DE MARLEN:



$$(a)^2+(c)^2 = (b)^2+(d)^2$$

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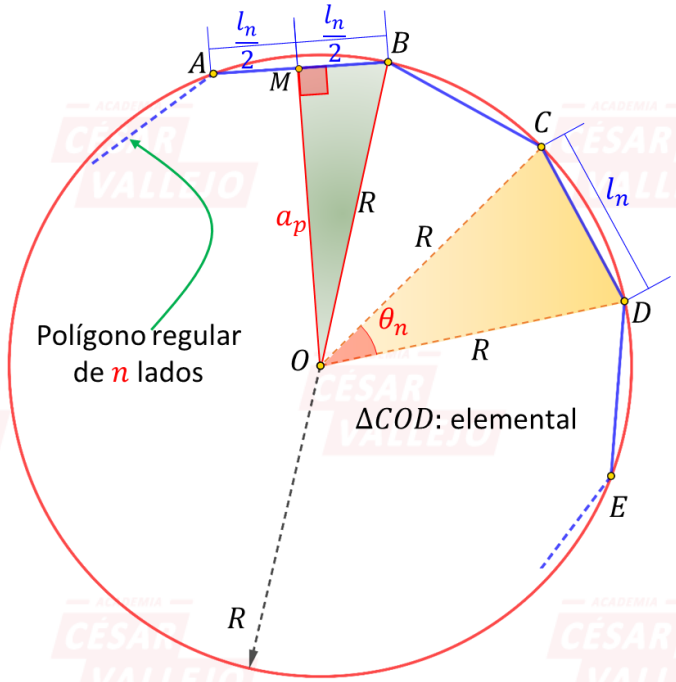
POLÍGONOS REGULARES

CÁLCULO DE LA LONGITUD DEL LADO Y APOTEMA DE UN POLÍGONO REGULAR



❑ **Cálculo de la longitud del lado,**

$$l_n = R\sqrt{2(1-cos\theta_n)}$$

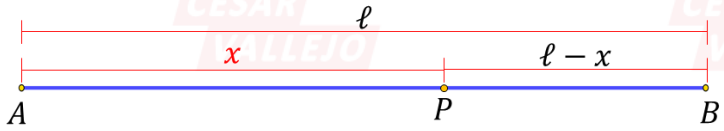
❑ **Cálculo de la longitud de la apotema,**

$$a_p = \frac{1}{2}\sqrt{4R^2-(l_n)^2}$$

$$a_p = \frac{R}{2}\sqrt{2(1+cos\theta_n)}$$

Sección aurea

Dado el segmento $\overline{AB}$, ubicamos P en $\overline{AB}$ $(AP > PB)$ P divide en media y extrema razón a $\overline{AB}$, cuando $\overline{AP}$ sea media proporcional entre $\overline{AB}$ y $\overline{PB}$.



De lo anterior:

$$\frac{AB}{AP} = \frac{AP}{PB} \rightarrow \frac{\ell}{x} = \frac{x}{\ell - x}$$

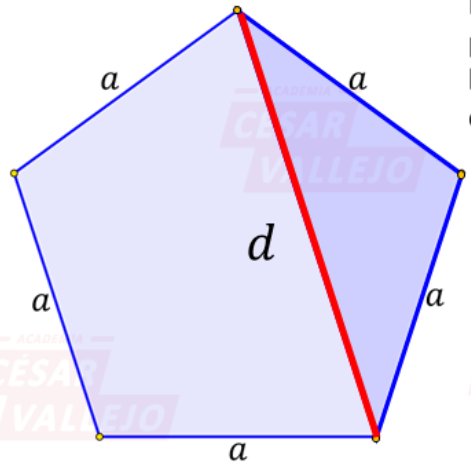
Operando:

$$x^2 + \ell x - \ell^2 = 0$$

$$x = \ell\left(\frac{\sqrt{5}-1}{2}\right)$$

- De la relación encontrada, indicamos que $\overline{AP}$ ***es la sección áurea de*** $\overline{AB}$.

PENTÁGONO REGULAR



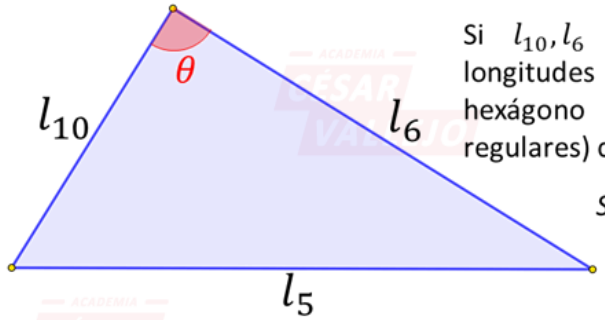
La longitud del lado de un pentágono regular es igual a la longitud de la sección áurea de su diagonal.

De manera práctica:

$$\frac{a}{d} = \frac{\sqrt{5}-1}{2}$$

También:

$$d = a\left(\frac{\sqrt{5}+1}{2}\right)$$



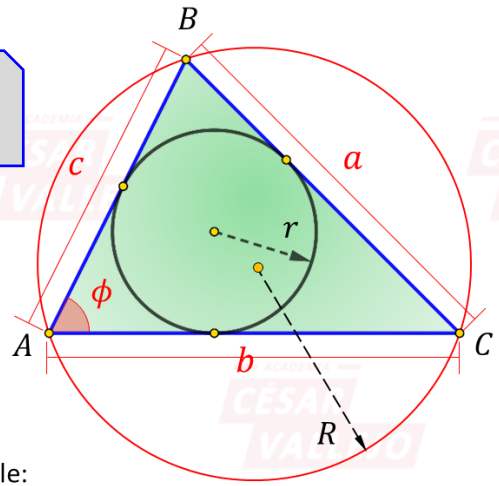
Si l_{10}, l_6 y l_5 representan las longitudes de los lados del decágono, hexágono y pentágono (todos regulares) con el mismo circunradio.

Se cumple: $\theta = 90°$

$$\leftrightarrow (l_{10})^2 + (l_6)^2 = (l_5)^2$$

ÁREAS DE REGIONES PLANAS I

REGIÓN TRIANGULAR



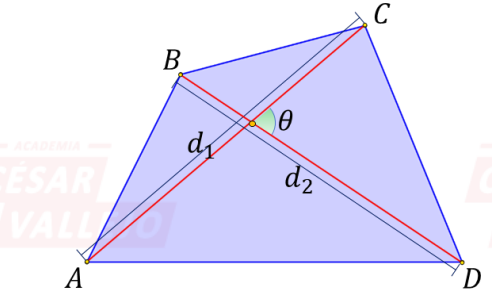
Se cumple:

$$\mathbb{A}_{ABC} = \frac{b \cdot c}{2}(sen\phi) \qquad \mathbb{A}_{ABC} = \frac{a \cdot b \cdot c}{4R}$$

$$\mathbb{A}_{ABC} = p \cdot r \qquad \mathbb{A}_{ABC} = \sqrt{p(p-a)(p-b)(p-c)}$$

Donde: $p = \frac{a+b+c}{2}$

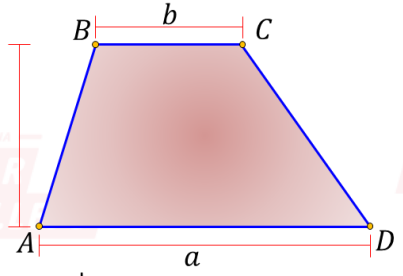
REGIÓN CUADRANGULAR



Se cumple:

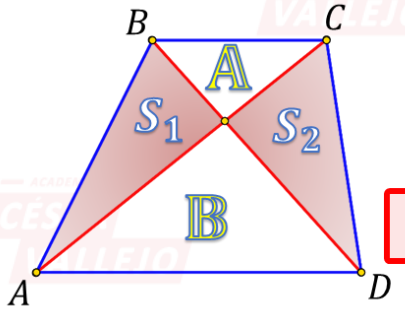
$$\mathbb{A}_{\square ABCD} = \frac{(d_1)(d_2)}{2} sen\theta$$

$ABCD$ es un trapecio de bases $\overline{AD}$ y $\overline{BC}$



Se cumple:

$$\mathbb{A}_{\square ABCD} = \left(\frac{a+b}{2}\right) \cdot (h)$$

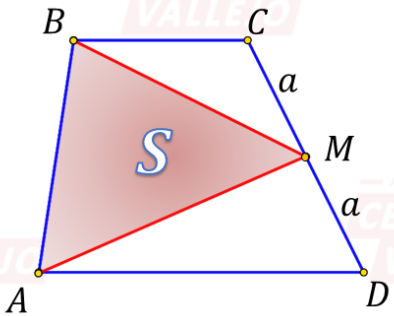


Se cumple:

$$S_1 = S_2$$

Además:

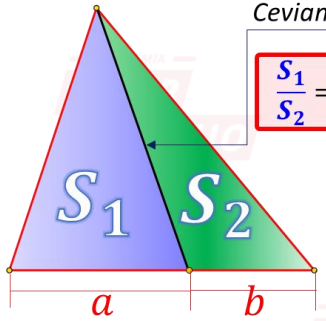
$$S_1 = \sqrt{\mathbb{A} \cdot \mathbb{B}}$$



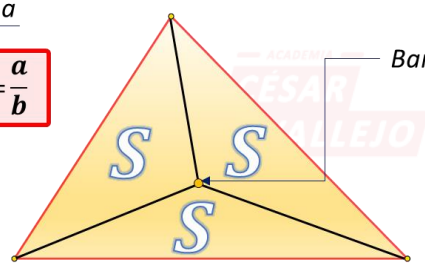
Si $CM = MD$

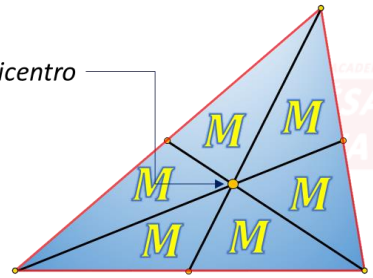
Se cumple:

$$S = \frac{\mathbb{A}_{\square}}{2}$$

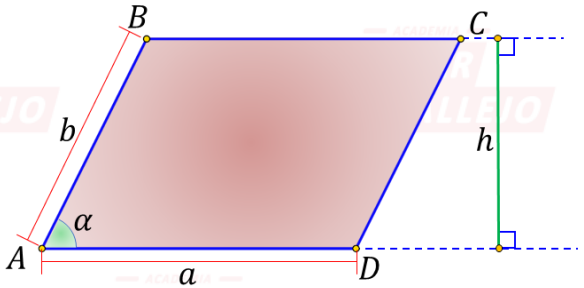


$$\frac{S_1}{S_2} = \frac{a}{b}$$





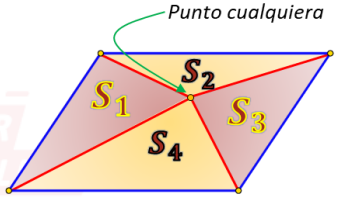
$ABCD$ es un paralelogramo



Se cumple:

$$\mathbb{A}_{\square ABCD} = a \cdot h$$

$$\mathbb{A}_{\square ABCD} = a \cdot b \cdot sen\alpha$$



Se cumple:

$$S_1 + S_3 = S_2 + S_4 = \frac{\mathbb{A}_{\square}}{2}$$

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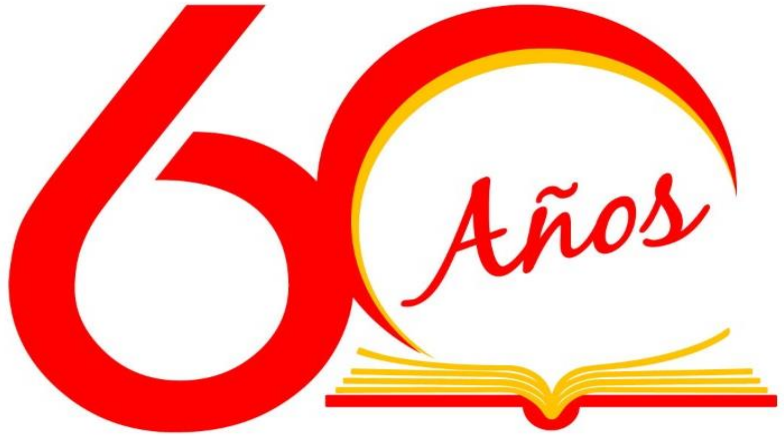

¡Gracias!